A REVIEW OF FRICTION DAMPING OF TURBINE BLADE VIBRATION

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(NASA-CR-187938) A REVIEW OF FRICTION DAMPING OF TURBINE BLADE VIBRATION (Carnegie-Mellon Univ.) 22 p N91-70438

Unclas
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1. INTRODUCTION

Service failures of gas-turbine blades can be attributed in many cases to high cycle fatigue caused by large resonant stresses. To avoid such failures, designers of aircraft engines frequently incorporate friction devices into turbine designs in order to increase damping and reduce vibratory stresses. These devices tend to be of two types: *blade-to-ground* (B-G) dampers that provide a link between a vibrating point on the blade and a relatively rigid structure such as a coverplate and *blade-to-blade* (B-B) dampers that provide a link between neighboring blades. In both cases the damper transmits a load through a friction contact which dissipates energy when slip occurs. One form of B-B dampers are seals that fit between the blades and rest on the underside of the blade platforms (see Fig. 1). This is the most popular type of damping device since it increases turbine efficiency as well as reduces the vibratory stress of the blades.

In order to design a friction damper, its geometry and the geometry of the contact region where it contacts the blade must be determined. To do this the design engineer must be able to calculate how changing the damper/blade system will affect its resonant response and then choose a design which will reduce the peak response as much as possible. A friction constraint may either stick or slip depending on the magnitude of the vibratory motion and, as a result the analysis of friction damping is a nonlinear problem. A significant amount of research on modelling friction constraints, of their effect on turbine blade response and how to optimize their design has been documented in the literature. The purpose of this paper is to primarily review the work reported during the 1980's

with a view towards establishing the current state-of-the-art. In the process the author has concentrated on reviewing research with which he is most familiar and which he feels is most relevant to turbine blade friction damper analysis and design.

The papers will be discussed in the following context: previous literature reviews, methods of analysis, models of friction constraint, methods of modelling the structure, methods for optimizing the damper's design, some related work and, lastly, some conclusions.

2. PREVIOUS LITERATURE REVIEWS

All of the authors that work in friction damping review the literature to some extent in their papers. However, often, they concentrate on only those papers which have some direct bearing on the direction of their own research and, consequently, do not provide an overview of the field. The papers reported in this section, however, provide a more general review of the field.

Ungar reviews the status of engineering knowledge concerning the damping of built-up structures as of 1973 in [1]. He describes the importance of structural damping, various damping mechanisms and concludes that in particular additional work is required on high-frequency damping of built-up beams and the low-frequency response of skin-stringer configurations.

Jones in a series of articles reviews high temperature damping methods [2-5]. The first of these articles appears in 1976 and the last in 1985. One part of the reviews pertain to friction damping. In his 1976 review he refers to a number of papers as developing "rational mathematical models of the essentially nonlinear behavior of highly idealized systems and the development of techniques for obtaining solutions." He concludes that, "these analyses are of general interest but are very difficult to apply to specific engineering problems." He updates the results of his initial paper in 1979 [3], in 1982 [4] and in 1985 [5]. It is clear from the number of papers cited in his reviews that research in friction damping became significantly more popular after 1982.

Rieger published a review paper on the damping properties of turbine blades in 1979 [6]. He made the following conclusion, "no meaningful

technical development has occurred in the area of steam turbine blade damping for at least the past 20 years".

Plunkett reviewed the status of friction damping as of 1979 in [7]. He discusses the relative roles of macroslip and microslip in damping vibratory response. He states that microslip is important at high contact loads or when contact is distributed over a relatively large area. He concludes that energy dissipation due to microslip "increases like the third power of the displacement and coulomb friction can control resonant vibration amplitude for excitation levels less than that necessary to cause macroslip." He also states that macroslip "is very effective in controlling resonant amplitude for those exciting forces less than some critical value." He cites some analytical and experimental results for some specific cases.

Beards has reviewed damping in structural joints on several occasions [8-11]. In his 1982 paper [9], he states that "the inherent damping of fabricated structures can be greatly increased without impairing the integrity of the structure by controlling the clamping forces in joints. Furthermore, the frequencies at which resonance occurs can be controlled." This last comment is especially important in turbine blade vibration since the state of the friction constraint can effect the blade crossing frequencies. This is also the theme of his 1983 review paper [10]. Beards also finds that the number of papers published on friction damping significantly increased by 1985 [11]. This work of the mid-80s will be discussed in some detail in the next sections.

3. METHODS OF ANALYSIS

Friction problems are nonlinear and, consequently, difficult to analyze exactly. Den Hartog [12] obtained the exact steady-state response of a single degree-of-freedom system with rigid/perfectly-plastic friction damping, subject to harmonic excitation. He utilized the piecewise linear nature of the equation of motion, the symmetry of the steady-state response within each period, and additional physical argument to obtain a system of equations and unknowns which could be solved analytically for the steady-state (long time, constant amplitude) response. This approach, however, quickly becomes impractical when faced with

more complicated systems, i.e. multiple nonlinearities or multiple degrees-of-freedom.

Because friction constraints are frequently modelled as piecewise linear (either the contact is stuck and the relative displacement is fixed, or the contact slips and the force is known) friction damping problems have been solved by piecing together linear solutions to obtain transient response curves, see for example [13]. In practice this is a difficult approach to implement if one is interested in the long time solution because the solution trajectory must be monitored closely to determine when to switch from one linear regime to another, and to establish the correct initial conditions for the new regime. In practice, this method requires numerical implementation. This approach also becomes impractical for multiple degrees-of-freedom systems since it becomes increasing difficult to discern all the regimes of motion. Consequently, if a transient solution is required it is easier and more efficient to use standard, finite difference based, numerical integration techniques.

Numerical integration readily admits multiple degrees-of freedom and nonlinearities. A number of text books are available which describe popular methods for solving systems of differential equations, see for example [14]. Because numerical integration tracks the time history of the solution, it can obtain the values of the nonlinear forces at each step from the present and past states of the system, which are readily available. If it is desired to employ higher order numerical integration methods then care must be taken to isolate the slip-stick transition in the friction constraint. This is usually done by taking smaller time steps across the transition point or by having the transition point occur at approximately the end of a time step. Otherwise, the accuracy of the higher order method degenerates and the procedure becomes extremely inefficient. The disadvantage of numerical integration is the time and computational effort required to find long-time or steady-state solutions, especially for lightly damped systems such as turbine blades. The method also requires some condition for determining that steady-state has been achieved, since the duration of the transient is indeterminate a priori. The computational costs can become prohibitive when many simulations are desired (for example, in establishing the optimum weight of a friction damper). For this reason, time integration solutions are most frequently used to check the accuracy of analytically determined approximate solutions.

One of the most popular methods for approximating the frequency response of nonlinear systems is known as the Harmonic Balance (HB) method in the vibration literature (see, for example, [15]), and as the Describing Function approach in the control literature (see, for example, [16]). Other methods that are used are known as the Equivalent Energy Balance method, Equivalent Linearization, the method of Slowly Varying Parameters, as well as, various perturbation methods. Iwan [17] shows that if these methods are employed correctly then, to first order, they all yield the same steady-state solution when applied to a frictionally damped system. (The method of Slowly Varying Parameters can also be used to yield the stability of a steady-state solution).

The HB method assumes that the excitation is harmonic and that the system response is also harmonic, with the same frequency as the excitation, but with an initially unknown amplitude and phase. Using the assumed form of the solution, the nonlinear force can be calculated for one period of oscillation in terms of the unknown amplitude and phase. The nonlinear force is expanded in a Fourier series and only the fundamental harmonic is kept. This allows the nonlinear force to be represented as a transfer function -- with a gain and phase shift -- which depends on the amplitude of the response. Combining this linearization of the nonlinear force with the assumed form of the solution reduces the equation of motion to a pair of nonlinear algebraic equations which are solved to determine the the amplitude and phase of the response. The nonlinear algebraic equations are solved using standard numerical methods, see, for example, [14]. When this approach can be applied, and trusted to give accurate solutions (where a single harmonic description of the nonlinearity is adequate), it is by far the most computationally efficient method for obtaining steady-state solutions to nonlinear dynamic problems. Typically, solutions are developed iteratively and the methods converge more quickly if you use a good initial guess. Good initial guesses are readily made in damper optimization studies since you may start with a linear problem (e.g., a locked joint) and vary a key parameter such as joint normal load to see its affect on system response. Consequently, the steady-state solution for one set of conditions can be used as a good initial guess to the solution at a neighboring state. As a result, solutions calculated using the Harmonic Balance approach usually require significantly less computational time than those calculated using time integration.

The HB method assumption that the response can be approximated by a single harmonic component has been verified in a number of studies in which HB solutions have been compared with solutions from time integration or from analog computers, e.g., [17,18,19]. The only time that the method tends to break down appears to be when additional frequencies are important. This may be the case when the friction element is rigid and the friction constraint is stuck for a significant portion of each cycle [20], or, when the higher frequency harmonics in the nonlinear force are important, e.g., if there is a resonant frequency at an integer multiple of the fundamental excitation frequency [21]. For the case of interest here, optimizing friction damping so as to minimize the peak response of turbine blades, these exceptional cases are usually not relevant and the HB method can be used to give an efficient solution to the problem which is sufficiently accurate.

4. MODELS OF FRICTION CONSTRAINT

Various approaches have been used to model the friction constraint in vibration calculations. By far the most frequently used model is that attributed to Coulomb which idealizes slip as taking place at a point. The constraint at the point is such that either no slip occurs or if slip occurs then the friction force is equal to a constant (the coefficient of friction) multiplied by the load component normal to the surface. The assumption is sometimes referred to as the "macroslip" assumption in that either the contact point completely slips or locks up. Coulomb models of point contact have been used in friction damping analyses, for example, by Griffin in [22], Muszynska and Jones in [23], and Dowell and Schwartz in [24]. In general, researchers have found that this simple model has worked relatively well as indicated by their ability to correlate their theoretical predictions with experimental data over a wide range of conditions, for examples see [22], [25], [26], and [27].

Menq, et al. investigated the effect of variable normal load on damper performance [28]. The normal load at the friction contact point can vary dynamically with the motion if the sliding surface is inclined with respect to the direction of motion. It was shown that variable normal load has a detrimental effect on damper performance. Consequently, one would conclude that dampers should contact the blade in such a way that the contact surfaces are parallel to the direction of vibratory motion. Earles and Williams

also consider the effect of a varying normal load in their model of shroud contact in [29].

Sinha and Griffin analyzed the effect of having a higher static coefficient of friction and a lower dynamic value on damper performance in [30]. They found that the static coefficient of friction had little effect on dynamic response except at high normal loads. At high normal loads their results may not be applicable since they did not take into account microslip.

Microslip refers to the idea that the friction contact takes place over a finite area rather than at a point and, because of the non-uniform nature of the stresses on the friction interface, that only part of that area may be slipping while the rest remains stuck. Microslip becomes important under conditions of high normal loads or when the friction contact region is large. Menq, et al. developed a microslip model of contact in [31] and used it to explain turbine blade friction damping data and shroud damping data in [32]. They found that a significant amount of friction damping can occur at high normal loads for which macroslip models would predict lock-up. These results indicate that friction dampers may be effective over a far broader range than would be indicated by macroslip analyses. Other researchers have used microslip to explain the response of built-up beams in [33], [34] and [35]. Goodman and Krumpp consider microslip in their analysis of root damping of turbine blades in [36].

Recently, Srinivasan and Cassenti have presented the idea of using a nonlocal friction law to model friction contact [37]. Their paper follows from Oden's work on modelling contact problems in elasticity [38]. One effect of such an approach is that the nonlocal law provides a more gradual transition from stick to slip than a simple coulomb model, in a manner similar to microslip. There is a computational advantage in smoothing the nonlinearity, in that the effective order of finite difference based methods is limited by the continuity of the solution (refer to Section 2). Since the second derivative of the solution exhibits the same smoothness as the forces, smoothing the nonlinear force improves the continuity of the solution and increases the effectiveness of time integration methods.

Experimental approaches have also been used to characterize the nonlinear constraints that can occur in joints (including friction and gap effects). For example, Crawley and Aubert [39] directly measure

joint response (for a joint used in a space truss) and describe the joint force in terms of a force-state mapping technique.

5. STRUCTURAL MODELS

This section reviews the types of structural models that have been used to represent turbine blade/friction damper (TB/FD) systems. These structural models are then integrated with an harmonic balance analysis in order to calculate steady-state response. If one thinks of the blade in terms of a single mode model then Den Hartog's analysis of how friction damps the forced response of a vibrating mass is relevant to this review [12]. In Den Hartog's model the mass rests directly on a rigid surface which transmits a friction force to the mass. Because of the rigid constraint, the mass either slips and moves, or is stuck and is at rest. Clearly, if this model represented a turbine blade/damper system, then it would be easy to optimize the damper's performance by increasing the normal load until the mass is stuck all of the time. In actuality this approach provides an accurate representation of a TB/FD system for only low normal loads since it does not simulate the flexibility of the damper element or the blade. For high normal loads the friction contact point will lock-up and, yet, the blade still vibrates in a "stuck" mode (the mode the blade would assume if the damper acts as a spring restraining the blade platform). Consequently, for high normal loads the damper is stuck, there is no friction damping in the system, and the vibratory stresses are not minimized.

Griffin considered the effect of the damper's flexibility in [22]. His model is essentially the same as Den Hartog's with the addition that the friction constraint is in series with a linear spring which represents the modal stiffness of the damper. This model can represent a wider range of normal loads than Den Hartog's since at lock-up the system continues to vibrate because of the damper spring. This model represents the TB/FD system reasonably well if the blade's mode shape is not significantly affected when the damper spring is pinned under high normal load conditions. Griffin suggested that mode shape change, or the blade's flexibility, could be taken into account approximately by "interpolating the values of modal parameters between their stuck and free values in a manner dependent upon the amount of slip present". If no such interpolation is used, Menq and Griffin [19] showed that the single mode model could lead to significant errors when the modal stiffness of the

damper was comparable to the modal stiffness of the blade (very stiff damper and a flexible extended neck on the blade, or the damper located in a very outboard location on the blade). They showed that under these conditions a receptance approach could be used to model the blade. Subsequently, they used the receptance approach to model TB/FD systems as well as a blade with a frictionally constrained part span shroud in [32] and [40]. They recommend that the receptances be calculated using finite element analyses with the results that their model gives results that are identical with those calculated using the finite element method when the system is linear, i.e. when the damper is fully slipping or locked, and closely approximates the results of time integration for intermediate conditions where friction damping is important.

Griffin and Sinha show that the forced response of a tuned bladed disk assembly with blade-to-blade dampers is mathematically equivalent to that of a single blade with a blade-to-ground damper in [41]. Consequently, with minor modifications, the models that have been developed to calculate single blade dynamics can also be used to analyzed the response of a tuned bladed disk assembly.

Muszynska and Jones simulate blade flexibility in their models by using multiple masses and springs to represent the blade [23], [42], [43], and [44]. They use both a two mass per blade model and a four mass per blade model in their research. The latter model allows the simulation of torsion/bending interaction. In both models the lower mass represents the blade platform where the damper contacts the blade. Consequently, when the lower mass locks-up the higher mass (representing the airfoil) may still vibrate. While their model considers blade flexibility it represents the friction damper as a rigid element. Consequently, their model is appropriate for either low normal loads (as is the case for the Den Hartog's analysis) or if the blades modal stiffness is much less than that of the damper. When this is not the case then the use of a rigid damper element tends to over predict damper performance.

A unique aspect of the Muszynska and Jones model's is that it represents multiple blades and can be used to model an entire bladed disk assembly. Consequently, they use their code to look at the effect of stage mistuning on friction damping [23]. Griffin and Sinha used a cruder model of multiple blades to investigate the effect of stage mistuning on damper performance and optimization in [41].

Dowell and Schwartz have analyzed the forced response of a cantilever beam with a dry friction damper attached in [24] using an approach involving Lagrange multipliers and the unrestrained beam modes. They compare their analytical predictions with experimental results in [25]. The damper is modelled as a massless spring in series with a Coulomb friction constraint and acts in a manner that resists transverse vibration of the beam.

Recently, Dowell has analyzed friction damping of beams and plates due to slipping at the support boundaries. In this case, in addition to transverse deflections, the coupled axial motion of the vibrating plate is considered. As the plate vibrates the tip of the plate expands and contracts in the length-wise direction and dissipates energy due to the friction constraint at the plate tip. This mechanism could explain friction damping in certain types of tip shrouded blades.

6. DAMPER OPTIMIZATION

A number of the papers that have been discussed in the last three sections have a bearing on selecting the optimum choice of damper parameters. In these models the damper is described in terms of its stiffness, the force required for slip at the contact point, whether its a blade-to-ground (B-G) or a blade-to-blade (B-B) damper, and the location of the point of contact on the blade.

Griffin shows both analytically and experimentally that damper stiffness is important for B-G dampers in [22]. According to his calculations the greater the modal stiffness of the damper the better it works in reducing vibratory stress. The modal stiffness of the damper is proportional to the physical stiffness of the damper multiplied by the modal displacement of the blade at the damper contact point, squared. Thus, one would concluded from this work that it is important to increase the physical stiffness of the damper as well as have it contact the blade at a point where the blade has as much deflection as possible. Once the damper stiffness is determined, Griffin gives simple expressions for calculating the optimum friction force in the joint and the resulting reduction in peak vibratory response. Griffin and Sinha extent this approach to blade-to-blade dampers in [41] where they give simple formulas for taking into account the interblade phase angle when the system is tuned. In [41] they consider tuned and mistuned bladed disks and find

that the friction slip force which optimizes the response of the tuned system approximately optimizes the response of the mistuned system as well. Consequently, their data suggests that friction dampers that are optimized under the assumption of tuned system response should work reasonably well for mistuned systems as well.

There are several limitations associated with the approach [22,41] for optimizing turbine blade friction dampers. One is that it does not take into account the flexibility of the blade and the change in the mode shape of the blade that may result from the friction damper constraint. Also, it does not take into account microslip. (Note: microslip usually provides additional damping, consequently, ignoring microslip should provide a more conservative estimate of damper response). Lastly, because the response of a frictionally damped turbine blade is nonlinear this approach requires that you know the amount of damping from other sources and the level of excitation acting on the bladed disk.

Muszynska and Jones develop analytical expressions for selecting the friction slip load for a tuned bladed disk assembly in [23]. The resulting equations for optimum slip load take into account the magnitude of the excitation, system damping, and blade properties as reflected in their multiple mass spring model of the blade. Because multiple masses and springs are used in their model their results includes the effect of the blade's global flexibility in that it allows the blade's mode shape to change for high damper forces. However, they represent the damper as a rigid link between blades and, consequently, do not consider the effect of the damper's flexibility. Thus, their results are applicable when the modal stiffness of the blade is significantly less than the modal stiffness of the damper. The effect of this assumption is that the analysis may tend to overestimate the effectiveness of the damper in reducing peak response. For example, when Dominic applies the Muszynska and Jones code to perform analyses of friction dampers for the HPFTP of the SSME engine he concludes, based on his analyses, that "The response amplitude of the airfoil is reduced more than two orders of magnitude at the minimum response in the frequency transition region," [46]. The author of this review is not aware of any experimental data that indicates turbine blade friction dampers can be this effective in reducing blade response and it seems likely that this conclusion would not have been reached if the effect of damper's (and local blade) flexibility had been included.

Muszynska and Jones also examine the effect of mistuning on selection of an optimum damper slip force and find [42] that the slip load which optimally damps a tuned bladed disk also works well for a mistuned disk. This is consistent with the previously cited results of [41] even though very different mass/spring models were used to represent the bladed disk assembly in the two studies. If this result holds in general, then it means that the problems associated with analyzing mistuning and with designing friction dampers are decoupled and can be analyzed separately. And, as a result, the damper designer can concentrate on analyzing the tuned system which, in turn, is mathematically equivalent to a single blade with a blade-to-ground damper.

Cameron, et al. discuss an integrated approach to turbine blade friction damper design in [47]. Their contention is that in designing a friction damper a variety of complications may occur which are not easily anticipated. As a result, they recommend that the analytical design and optimization of the damper's performance be carried out in conjunction with an experimental program consisting of bench tests on blades with blade-to-ground dampers. Initially the dampers are designed with an analytical model calibrated from past experience. They recommend that the effective stiffness of the damper and coefficient of friction be chosen so that the analytical model best fits the experimental results. It is also proposed that the validity of the analytical model (i.e. whether microslip is important, does the structural model adequately represent the blade, are variable normal load effects important, etc.) be checked by examining the model's ability to match the test data over a wide range of input frequencies, input force magnitudes, and damper loads. (If it does not match the test data adequately, then an improved model should be developed). Once the damper model has been calibrated and tested in this manner, it is then used to select the final design parameters so as to optimize the damper's performance for engine operating conditions. The optimization process is not always straight forward in practice since the stiffness of the damper and its mass (which controls the damper slip load) are integrally related though its geometry. Lastly, Cameron, et al. propose an approach for optimizing damper performance which does not require that the magnitude of the excitation or the amount of damping in the engine be known a priori. Consequently, it is an easier approach to employ for new turbine designs in which the excitation acting on the stage is not known

from previous engine tests. The approach is applied by Kielb, et al. to the design of a high performance turbo-pump in [48].

7. SOME RELATED WORK

A study of turbojet engine blade damping is reported by Srinivasan, et al. in [49]. They found that attachment damping (due to rub in the dovetail) was probably insignificant at operational speeds because of the high contact pressures. They found that material damping was insignificant for the materials tested. On the other hand they found that friction damping could strongly affect vibratory response in the case of part-span shrouds. Lastly, they conducted tests on turbine blade platform dampers but the results where not conclusive as it appears that they did not test over a sufficiently wide range of normal loads so as to establish the optimum damper design point.

Zmitrowicz reports the a finite element analysis of a turbine blade system damped by dry friction forces in [50]. He proceeds to discuss his formulation and gives some results of specific calculations. He does not discuss the optimization of the friction constraint.

Dowell analyzes the response of a pinned-pinned, frictionally damped beam in [51]. The mathematical approach is interesting in that a component mode analysis is carried out based upon the use of constraint conditions and Lagrange multipliers. The resulting formulation automatically takes into account changes in mode shape that occurs due to the friction constraint.

Various studies have been concerned with modelling the friction constraint at shroud interfaces. The goals in designing shrouds are somewhat different from those of friction damper design in that shrouds are usually designed with high normal loads in order to provide a constraint which restrains motion. Consequently, microslip and variable normal load effects tend to be important. Bielawa analyses the effect of microslip on blade response for the case of inter-shroud rubbing in [52]. Williams and Earles considered optimizing the placement of a part-span shroud in [53] and formulated their approach in [54]. Menq, et al. developed an approach for analyzing shroud constraints in [55].

Most of the work that has been done on using friction to control the vibratory response of turbine blades has considered a single

frequency, sinusoidal excitation. Sinha recently considered how friction damping could be used to reduce the vibratory response of a turbine blade subject to a Gaussian white noise excitation [56]. For example, this type of excitation could cause "buffet stresses" in the blades. Sinha found that the expected value of the square of the amplitude could be minimized as a function of slip load in an analogous manner to that developed for sinusoidal excitations in [22]. His method of analysis followed that used by Asano and Iwan in [57].

8. CONCLUSIONS

A significant amount of work on the friction damping of turbine blades has been conducted during the last ten years which has identified the potential benefits of optimizing friction damping and has identified some of the mechanisms that can affect the damper's performance and that are difficult to predict a priori. Because these mechanisms can sometimes strongly affect the dynamic response of the system it has been suggested that laboratory tests be used during the design phase to assess their importance and to confirm the validity of the analytical model being used. Such checks are required because a definitive model of friction damping which predicts damper performance under all circumstances does not presently exist. The lack of a complete model of turbine blade friction damping not only requires that expensive and time consuming tests be completed during the design phase but also limits the designer in terms of his understanding of how to fully exploit friction damping.

One important result of the recent research has been the observation that the response of a mistuned bladed disk assembly is minimized by the same choice of damper parameters that minimizes the resonant response of the system if it were tuned. The response of a tuned bladed disk can be characterized in terms of the response of a single blade and disk segment. Consequently, the magnitude of the required computations is typically reduced by two orders of magnitude. This means that future research effort can be directed towards developing improved models of single blade response (for example, they may include multiple points of contact between the blade and the damper, slip that follows an elliptical orbit across the friction interface, the incorporation of microslip effects, and improved modelling of mode shape changes) that are valid under a

broader range of conditions. If this is done then the design engineer will have a more complete design tool that may help inspire better damper designs and eliminate some of the current needs for laboratory tests.

ACKNOWLEDGEMENTS

This review was carried out as part of a research contract for Rockwell International which was sponsored by the NASA Marshall Space Flight Center under the direction of Mr. J. Cannon, NASA Contract # NAS8-36720.

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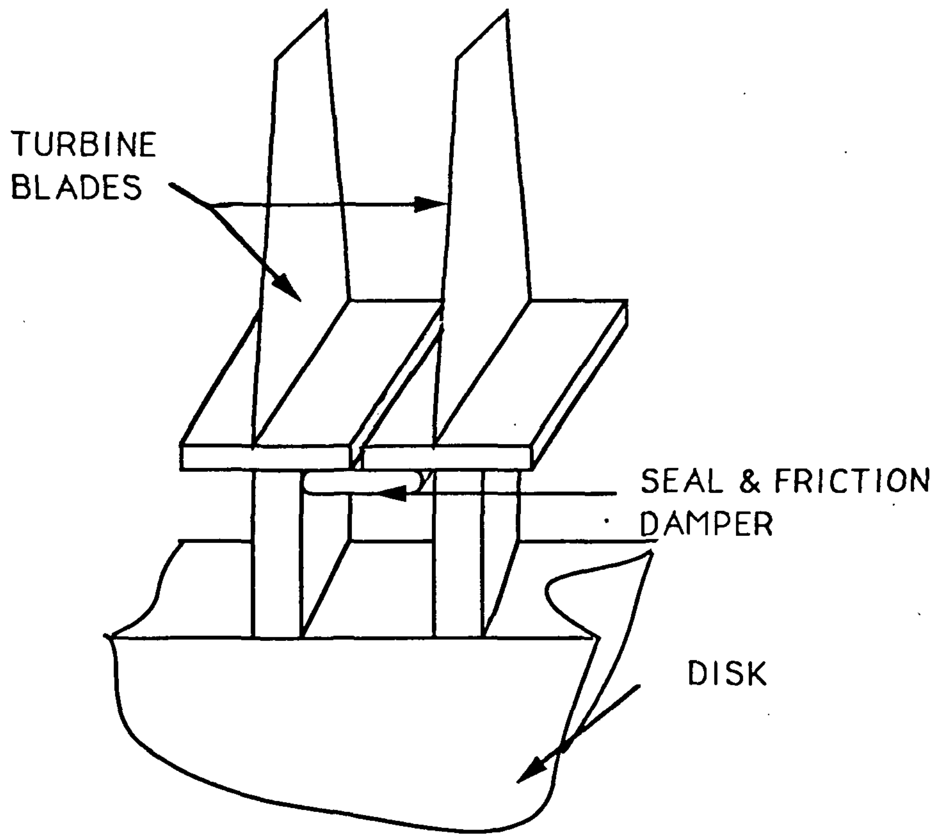
FIGURE CAPTIONS

FIGURE 1. SHEMATIC SHOWING LOCATION OF SEAL/FRICTION DAMPER


TURBINE
BLADES
SEAL & FRICTION
DAMPER
DISK